Lift of Delta Wings with Leading-Edge Blowing

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An analysis of the lift augmentation due to a thin jet of air issuing from a slot along the leading edge of a delta wing is presented. The problem is treated with an extension of the method of Brown and Michael, representing the separated flow on the leeward side of the wing by a pair of concentrated vortices and corresponding feeding sheets. It is assumed that the jet is not affected by Coanda forces. The analysis produces reasonable agreement with experiments and suggests ways of grouping the concept parameters.

Nomenclature

a	= wing semispan
C_μ	= jet momentum coefficient
e_j	= complex unit vector in direction of ejection
f,g	= universal functions in lift coefficient expressions
F	= resultant force on singularity system
F_s	= force on connecting vortex sheet
F_v	= force on vortex
i	= $\sqrt{-1}$
k	= constant in lift augmentation expression
m_j	= jet momentum flux per unit length
p	= exponent in lift augmentation expression
V_∞	= freestream velocity
v_d	= velocity at center of vortex
W	= complex potential in cross-flow plane
α	= angle of attack
β	= ejection angle
ϵ	= half-apex angle
Γ	= vortex intensity
ρ	= freestream fluid density
ρ_j	= jet fluid density
σ	= complex representation of physical cross-flow coordinates
θ	= complex representation of transformed coordinates
σ_0,θ_0	= vortex equilibrium location
$(\bar{\ })$	= complex conjugate

Introduction

A THIN jet of air ejecting from a slot along the leading edge of a delta wing alters the equilibrium position of the vorticity system on the leeward side of the wing, causing a change in the pressure distribution on the wing surface, thus resulting in lift augmentation. In addition to the pressure-induced lift increment, the lift is also enhanced by the vertical component of the jet momentum.

The jet of air may leave the wing surface in two different ways. It may exit in a direction uniquely determined by the orientation of the slot, or it may follow the wing surface and separate from it tangentially. In the first case, the jet direction is not affected by Coanda forces. The first case will occur if the jet exit direction, imparted by the slot, forms a sufficiently large angle with respect to the local tangent to the edge of the wing's cross-sectional cut (in the cross-flow plane). In this case, the jet affects the wing aerodynamics by injecting momentum into the vortex structure, which implies that significant blowing intensity is needed to alter the lift. The second case will occur if the direction of ejection makes a sufficiently small angle with respect to the local tangent direction. In this case, the jet will act on the lift by altering the position of the primary separation point, while the momentum of the jet injected into the vorticity system will be quite small.[1] This paper will consider the first case, where the direction of the jet is not affected by Coanda forces. The concept is illustrated in Fig. 1.

The problem in conical symmetry with blowing in the direction of the wing span was solved by Barsby,[2,3] who based his analysis on Smith's description[4] of the separated flow about a conical, flat delta wing. Experimental work has been reported by Trebble,[5] who attempted to simulate conical symmetry. Measurements on a cropped delta wing with this blowing concept have been reported by Alexander.[6,7]

This paper investigates the problem in a much simpler mathematical framework than that used by Barsby, and the angle of jet ejection with respect to the wing-span direction is included as an additional parameter.

The theory first proposed by Brown and Michael for the analysis of delta wings without blowing[8] is extended to account for momentum injection into the vortex system. In this approach, the separated flow on the wing is represented by a pair of vortices connected to the leading edges by straight vortex sheets, as shown in Fig. 2. Brown and Michael solved the no-blowing delta wing problem in the cross-flow plane by requiring that the forces, but not the moments, acting on the singularity system should be in equilibrium. This method leads to a complex-valued, implicit equation for the equilibrium position of the singularity system. Once the equilibrium position is established, the vortex and sheet intensities are determined from the tangency condition at the leading edge, and with this the lift is readily obtained.

The procedure developed here follows the same steps, except that the force-balance condition is altered to account for the momentum ejected from the wing at a given angle with the span. The implicit relationship for the equilibrium position of the singularity system in this case differs from that of Brown and Michael's in that it contains a source term proportional to the momentum intensity of the jet. The lift obtained from the development does not include the vertical component of the jet momentum.

Mathematical Model

To determine the condition for equilibrium of the singularity system, consider the cross-flow plane with the control volume shown in Fig. 3. The sum of all the forces acting on the singularity system must balance the momentum transfer through the volume walls. Assuming that the momentum associated with the jet aligns itself with the direction of the core within the control volume, and interpreting the forces as complex quanti-

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ties, momentum balance requires

$$F_s + F_v = -m_j e_j \tag{1}$$

Expressions for these forces were derived by Brown and Michael[8]

$$F_s = i\rho V_\infty \frac{d\Gamma}{dx}(\sigma_0 - a) \tag{2}$$

$$F_v = -i\rho\left(v_d - V_\infty \epsilon \frac{\sigma_0}{a}\right)\Gamma \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1)

$$v_d = V_\infty \epsilon\left(\frac{2\sigma_0}{a} - 1\right) - i\frac{m_j}{\rho\Gamma}e_j \tag{4}$$

An expression for v_d is obtained through the transformation $\theta = \sqrt{\sigma_0^2 - a^2}$, which maps the cross-flow plane into a plane where the wing is represented by a slot along the imaginary axis, as illustrated in Fig. 4.

$$\bar{v}_d = \lim_{\sigma \to \sigma_0}\left[\frac{dW}{d\theta}\frac{d\theta}{d\sigma} + \frac{i\Gamma}{2\pi(\sigma - \sigma_0)}\right] \tag{5}$$

$$W(\theta) = -\frac{i\Gamma}{2\pi}\log\frac{\theta - \theta_0}{\theta + \bar{\theta}_0} - iV_\infty \alpha\theta \tag{6}$$

The vortex intensity is obtained from the requirement that the velocity at the leading edges should be finite

$$\frac{2\pi V_\infty \alpha}{\Gamma} = \frac{1}{\theta_0} + \frac{1}{\bar{\theta}_0} \tag{7}$$

Introducing the jet momentum coefficient

$$C_\mu = \frac{2m_j}{\rho V_\infty^2 a} \tag{8}$$

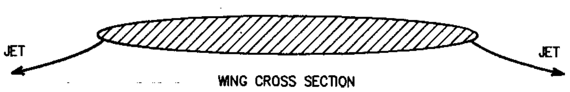

Fig. 1 Delta wing leading-edge blowing concept.

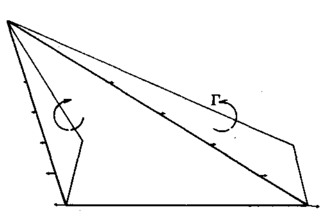

Fig. 2 Brown and Michael model.

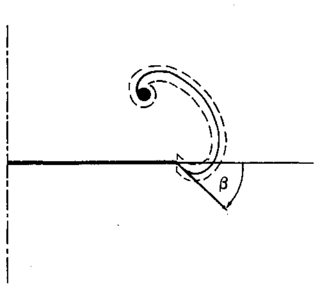

Fig. 3 Jet sheet control volume.

and carrying out the limit above, the equilibrium condition for the singularity system is

$$\left[\frac{1}{\theta_0^2 + \theta_0\bar{\theta}_0} - \frac{1}{\theta_0\bar{\theta}_0} - \frac{a^2 - 2\sigma_0^2}{2\sigma_0^2\theta_0^2}\right]\sigma_0 + i\frac{\epsilon}{\alpha}\left[\frac{1}{\theta_0} + \frac{1}{\bar{\theta}_0}\right]\left[\frac{2\bar{\sigma}_0}{a} - 1\right] = \frac{a}{4\pi}\frac{C_\mu}{\alpha^2}\left[\frac{1}{\theta_0} + \frac{1}{\bar{\theta}_0}\right]^2 e_j \tag{9}$$

The term on the right-hand side contains the blowing information. In the expression for the lift coefficient derived by Brown and Michael

$$C_L = \frac{4\pi}{a^2}\epsilon\alpha\theta_0\bar{\theta}_0 + 2\pi\alpha\epsilon \tag{10}$$

the first term on the right-hand side represents the vortex lift, a nonlinear function of α, and the second term represents the linear part of the lift, which would be produced by the wing in the attached flow case. To assess what form Eq. (10) will take in the case of blowing, it is noticed that, in Eq. (9), apex angle, angle of attack, and momentum coefficient appear in two groups, whereas the angle of blowing β appears through the definition of e_j. Since the product $\theta_0\bar{\theta}_0$ is obtained by solving Eq. (9), the lift coefficient will be of the form

$$C_L = 4\pi\,\epsilon\alpha f\left(\frac{\epsilon}{\alpha}, \frac{C_\mu}{\alpha^2}, \beta\right) + 2\pi\alpha\epsilon \tag{11}$$

In this expression, f represents a universal function of its arguments and is not given analytically, since no exact analytic solution of Eq. (9) is possible. For some limiting values of the arguments, however, analytical approximations to Eq. (11) are possible. Notice that the blowing information appears in the group C_μ/α^2. This dependence could be helpful in arranging the problem parameters when conducting experiments.

To obtain such limiting forms, consider first the approximation to the lift coefficient in the absence of blowing, as obtained through linearization of Eq. (9)[8]

$$\frac{C_L}{\epsilon^2} = \frac{2\pi\alpha}{\epsilon} + k(\alpha/\epsilon)^{5/3} \tag{12}$$

Hence, the condition that the no-blowing case should revert to Eq. (12) gives

$$\frac{C_L}{\epsilon^2} = \frac{2\pi\alpha}{\epsilon} + k_1(\alpha/\epsilon)^{5/3}[g(C_\mu/\alpha^2,\beta) + 1] \tag{13}$$

The lift movemennt is then

$$\frac{\Delta C_L}{\epsilon^2} = k_1(\alpha/\epsilon)^{5/3}[g(C_\mu/\alpha^2,\beta) + 1] \tag{14}$$

The form of g for small values of its argument will be considered in the case of $\beta = 0$. It can be shown that g is regular for $C_\mu/\alpha^2 \to 0$. Expanding g in C_μ/α^2, the following limiting form results, valid for small C_μ/α^2 and $\beta = 0$:

$$\Delta C_L = k(\epsilon/\alpha)^{1/3}C_\mu \tag{15}$$

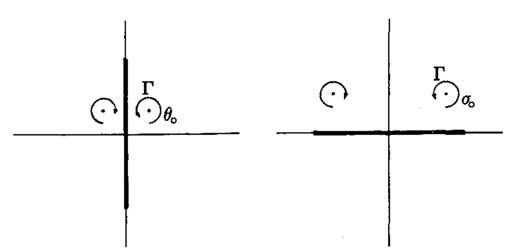

Fig. 4 Conformal transformation of cross-flow plane.

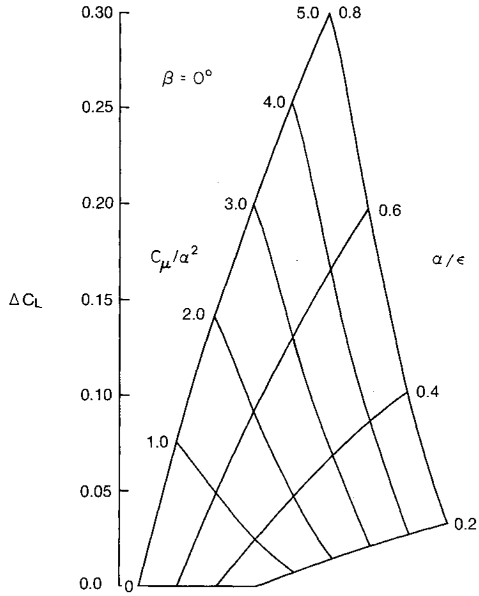

Fig. 5 Performance plot, $\epsilon = 20$ deg.

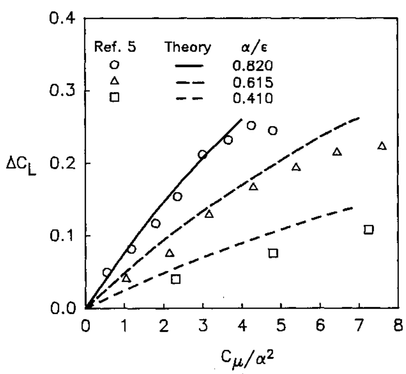

Fig. 6 Comparison of theory and experiment, $\epsilon = 20$ deg.

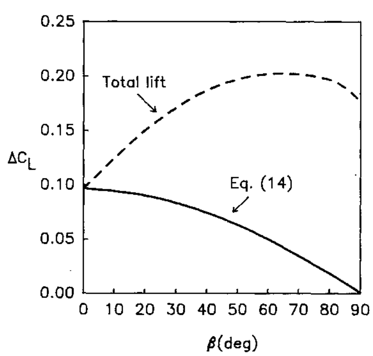

Fig. 7 Lift as function of ejection angle, $\epsilon = 20$ deg; $\alpha = 12$ deg, $C_\mu = 0.0875$.

The constraint imposed on blowing intensity in order for Eq. (14) to be applicable guarantees that no singularity at zero angle of attack will occur. Verification of Eq. (14) would require extremely small blowing intensities.

Results

Figure 5 shows, in carpet-plot form, the lift increment due to blowing in the spanwise direction.

Figure 6 presents a comparison of these calculations with the measurements taken by Trebble.[5] The theory somewhat overpredicts the lift increment, a feature also exhibited by Barsby's theory.

Figure 7 shows the lift produced by blowing at an angle to the wing's spanwise direction. The lift obtained from Eq. (13), which does not include the direct thrust of the vertical component of the jet momentum, is compared to the total lift, obtained by adding the vertical component of the jet momentum flux. A maximum occurs for ejection at about 60 deg.

Conclusions

The problem of blowing from the leading edges of a slender delta wing has been analyzed using a generalization of the vortex-connecting-sheet model for the separated flow on the leeward side of the wing. The results lead to the following observations.

Blowing from the leading edges of a slender delta wing causes an increment of lift beyond the vertical component of ejected momentum. Both theory and experiments suggest that, for wings of 40 deg apex angle, blowing coefficients of about 0.05 leads to gains in lift of the order of 30%.

The theory suggests a way of grouping the different nondimensional quantities of the problem in such a manner that the lift increment becomes a function of ϵ/α, C_μ/α^2, and β. It is expected that this particular way of grouping the wing and jet parameters will reduce considerably the size of the matrix of an experimental program.

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